Journal of Engineering Physics and Thermophysics, Vol. 76, No. 2, 2003

HEAT AND MASS EXCHANGE IN DEEP COOLING OF THE PRODUCTS OF COMBUSTION OF NATURAL GAS

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UDC 620.9:662.92; 658.264

The process of deep cooling of the products of combustion of natural gas with the condensation of a steam on the heat-exchange surface has been analyzed. The dependence of the relative volume concentration of the steam on the temperature of the gases in the process of cooling has been obtained.

Introduction. An increase in the cost of natural gas and a reduction in its supply (under the pretext of nonpayment) make it topical to cool combustion products below the dew point to obtain heat released in condensation of a steam and to utilize the condensate itself. The dew-point temperature of the combustion products of natural gas which escape from heating and steam boilers is usually 50 to 60$^{\text{o}}$C depending on the excess-air coefficient in them. There are examples (implemented in industry) of deep cooling of the gases escaping from boilers in both open-type [1] and surface [2–4] heat exchangers. In the first case, the combustion products escape from the heat exchangers, being saturated (and even "supersaturated" because of the removal of droplets) with steam; therefore, for condensation of the main part (at least 70%) of the steam contained in the combustion products one must cool these products in open-type heat exchangers down to approximately 30$^{\text{o}}$C, which can frequently prove economically inefficient and present problems of corrosion protection in by-passes and a smoke tube. Conversely, cooling of the gases in an open-type heat exchanger to 70 to 80$^{\text{o}}$C, as is sometimes done, leads to the evaporation of a part of the cooled water due to the heat of the escaping gases rather than to the condensation of the steam.

The situation is somewhat more attractive when one uses surface finned heat exchangers [2–4] in which the temperature of the wall can be 30$^{\text{o}}$C or lower (depending on the temperature of the cooling water) for rather high temperature of the combustion products.

Use of the Analogy of the Processes of Heat and Mass Exchange. Influence of the Stefan Flow. In calculating the quantity of the condensing steam (and accordingly the heat released due to this), one usually uses the analogy of the processes of heat and mass exchange. Indeed, the concentration field in the process of mass exchange not complicated by heat exchange and the temperature field in the process of heat exchange not complicated by mass exchange are described by the analogous differential equations since they result from the representations of the Fick law $j = -D\nabla\rho$ and the Fourier law $q = -\lambda\nabla t$ of the same form.

Virtually coincident are the representations of the boundary conditions on the wall for the mass exchange

$$j = \beta\,(\rho - \rho_{\text{w}}) \tag{1}$$

and the heat exchange

$$q = \alpha_{\text{conv}}\,(t - t_{\text{w}})\,. \tag{2}$$

But the process of mass exchange in condensation even in the isothermal case (for example, in chemical absorption of the steam on the wall) differs from the process of heat exchange by the presence of the Stefan flow, since the steam on the wall "disappears" in condensation. This process is analogous to the heat exchange with the suction of a part of the gas through a porous wall [5]. When the content of the steam in the combustion products of natural gas is low (no higher than 17% by volume), the influence of the Stefan flow proves insignificant.

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When the processes of heat and mass exchange occur simultaneously, strictly speaking, there is no analogy between them, if for no other reason than the mass concentration of the active material depends not only on the composition of the gas but also on the temperature. One can envision the hypothetical case where ρ_w is higher than ρ at $t_w < t$, despite the fact that the relative volume content r of the active material in the flow is higher than that at the surface r_w, i.e., the mass flux is directed toward the surface. In this case, representation in the form (1) proves incorrect qualitatively.

In combined heat and mass transfer, the material flux should be considered to be in proportion to the gradient of its chemical potential rather than the mass concentration, but this introduces substantial complications into engineering calculations, unjustified in the case of a weak influence of "cross" coefficients (of thermal diffusion and diffusion thermal conductivity).

In nonisothermal cases, it is recommended [6, 7] that the partial pressures of the active component be used instead of the mass concentrations in the equation of the Fick law, which (for isobaric processes) is equivalent to the relative volume concentration r. Then (1) will have the form

$$j = \beta_p \, (r - r_w) \, . \tag{1'}$$

When $r_w << r$ and $\rho_w << \rho$ and in the isothermal case (when $T_w = T$) formulas (1) and (1′) are equivalent; here

$$\beta_p = \beta \, \frac{p}{R_{st} T} \, , \tag{3}$$

where p will subsequently be taken to be equal to the "normal" pressure, i.e., 0.101 MPa.

In [6], Berman has obtained the empirical equations for the heat and mass exchange between the steam-gas mixture and the surface streamlined by it (in the notation adopted in this paper)

$$\frac{\text{Nu}_d}{\text{Nu}_{d1}} \, \Pi_d = \left(\frac{1 - r}{\Pi_d} - 0.4 \right)^{-1} \, , \tag{4}$$

$$\frac{\text{Nu}}{\text{Nu}_1} = 1 - 0.6 \left(\frac{1 - r}{\Pi_d} - 0.4 \right)^{-1} \, , \tag{5}$$

$$\Pi_d = r_w - r \, . \tag{6}$$

The volume fraction r of the steam in the combustion products of natural gas which escape from the boiler for an excess-air coefficient of 1.3 is approximately 0.15. If we take $r_w = 0.01$ (which corresponds to the saturated-steam pressure at $t_w = 7^{\text{o}}\text{C}$ for the combustion products and atmospheric pressure), we obtain $\text{Nu}/\text{Nu}_1 = 1.093$ from (5).

Thus, the Stefan flow related to the condensation of the steam increases α_{conv} by 10% at most at entry to the heat exchanger. With cooling of the gas and drying of the flow (with decrease in r) in the heat exchanger this effect will become weaker.

Analogously we obtain $\text{Nu}_d/\text{Nu}_{d1} = 1.104$ from (4), i.e., β increases by the same 10% at the beginning of the heat exchanger under the influence of the Stefan flow and nonisothermicity, but subsequently the influence of this flow also becomes weaker.

In [5], as a result of the experimental investigations of the mass transfer from the steam-gas mixture to a colder tube in cross flow, the following relation has been obtained (it holds for $(1 - r)/(r - r_w) > 1$):

$$\frac{\text{Nu}_d}{\text{Nu}_{d1}} = 0.71 \, (1 - r)^{-0.9} \, (r - r_w)^{-0.1} \, . \tag{7}$$

In the case in question we have $(1 - r)/(r - r_w)) = 6.07$ and $Nu_d/Nu_{d1}=1$.

The foregoing shows that in calculating the combined processes of heat and mass exchange between the combustion products containing no more than 20% of the steam and the cooling surfaces, one can calculate the convective coefficients α_{conv} and β without taking into account the mutual influence of the heat and mass exchange and, in particular, of the Stefan flow (at least accurate to 10%) and can determine the mass transfer by analogy with the heat transfer.

Calculation of the Heat Absorption of the Surface with Account for Condensation at a Prescribed Surface Temperature. We consider as an example heat and mass exchange for a tube in cross flow of steam-containing combustion products for Reynolds numbers of 10^3 to 10^5. In this range, the heat exchange not complicated by mass exchange is described as [8]

$$Nu_{conv} = 0.25\,Re^{0.6}\,Pr^{0.38}\,, \tag{8}$$

and the mass exchange is accordingly described by the equation

$$Nu_d = 0.25\,Re^{0.6}\,Pr_d^{0.38}\,, \tag{9}$$

where $Re = wd/\nu$, $Nu_{conv} = \alpha_{conv}d/\lambda$, $Pr_d = \nu/a$, and $Nu_d = \beta d/D$. The coefficient α_{conv} takes account only of the heat transferred to the heat-exchange surface by convective heat transfer:

$$q_{conv} = \alpha_{conv}\,(t - t_w)\,. \tag{10}$$

To q_{conv} there is added the heat released on the surface in condensation of the steam on it:

$$q_{cond} = \beta q_{st}\,(\rho - \rho_w)\,. \tag{11}$$

Dividing (9) by (8) and substituting the values of the corresponding numbers, we obtain

$$\beta = \alpha_{conv}\,\frac{D}{\lambda}\left(\frac{a}{D}\right)^{0.38}\,. \tag{12}$$

Then the total heat flux will be

$$q_\Sigma = q_{conv} + q_{cond} = \alpha_{conv}\left[(t - t_w) + \frac{D}{\lambda}\left(\frac{a}{D}\right)^{0.38} q_{st}\,(\rho - \rho_w)\right]. \tag{13}$$

The reduced (with allowance for the heat of condensation of the steam) coefficient of heat transfer

$$\alpha_\Sigma = \frac{q_\Sigma}{t - t_w} \tag{14}$$

increases with decrease in the temperature of the wall as the quantity of the steam condensing on it increases. For prescribed temperature of the wall and concentration of the steam in the flue gases, the fraction of the condensation component in the reduced coefficient of heat transfer is dependent neither on the velocity of the gases nor the diameter of the tube.

To check the calculations according to (13) we investigated heat transfer from the combustion products of natural gas at exit from the steam boiler in the USTU-UPI boiler house to a single water-cooled tube in cross flow [9]. The temperature of the cooling water varied from 30 to 60^oC. The temperatures of the escaping gases and of the dew point were equal respectively to 200 and 50.5^oC. The experimental tube with an outside diameter of 10 mm and a wall thickness of 1 mm was manufactured from stainless steel. Its thermal resistance is negligibly low as compared to that of the heat transfer from the gas. As the evaluations show, the thermal resistance of the formed condensate

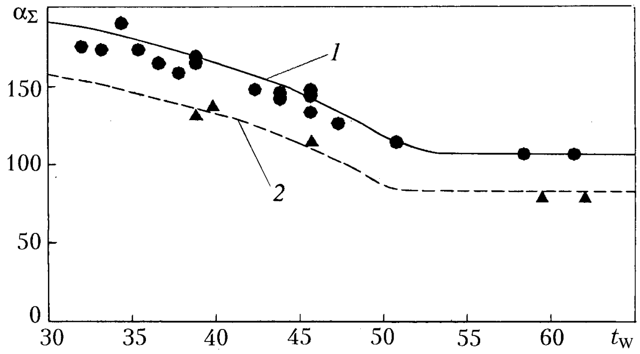

Fig. 1. Reduced coefficient of heat transfer vs. temperature of the wall of the heat-exchanger tube: curves) calculation; points) experiment [1) w = 11.3; 2) 7.7 m/sec]. α_Σ, W/(m^2·K); t_w, ^{o}C.

film is also insignificant; therefore, the temperature of the exterior tube surface was taken to be equal to the arithmetical mean of the temperatures of the cooling water.

Figure 1 shows that the results of calculation according to (8), (13), and (14) are in good agreement with experimental data. This confirms the applicability of the analogy of heat and mass exchange under these conditions and the absence of an appreciable influence of the Stefan flow.

Change in the Concentration of the Steam along the Cooling Surface with a Constant Temperature. Let us single out a differentially small portion in a channel with cooling surfaces (for the sake of clarity we will assume that these are channel walls with a temperature T_w, although this can be a tube bundle, including that manufactured from finned tubes). For this portion we write the equation of material balance of the steam:

$$dG_{st} = -\beta_p (r - r_w)\, dF\,. \tag{15}$$

The quantity β_p will be evaluated according to (3) with account for (12):

$$\beta_p = \frac{p}{R_{st}T}\alpha_{conv}\frac{D}{\lambda}\left(\frac{a}{D}\right)^{0.38} = \alpha_{conv}\frac{p}{R_{st}T}\left(\frac{D}{a}\right)^{0.62}\frac{1}{\rho_m c_p}\,. \tag{16}$$

The dependence of the coefficient of diffusion of the steam in air (in the temperature range 282–450 K of interest and at atmospheric pressure) has the form [10]

$$D = 0.205\cdot10^{-4}\left(\frac{T}{273}\right)^{2.072}. \tag{17}$$

For the coefficient of diffusion of the steam in air, the formula

$$D = 0.209\cdot10^{-4}\left(\frac{T}{273}\right)^{1.89}\frac{p_0}{p} \tag{18}$$

is recommended in [11]. The diffusion coefficients calculated from (17) and (18) differ by 1 to 6% ($p = p_0$ = 0.101 MPa in (18)); the ratio $(D/a)^{0.62}$ is equal to unity accurate to 10%. Taking $(D/a)^{0.62} = 1$ and replacing ρ_m by $r/(R_m T)$ in (16), we obtain

$$\beta_p = \alpha_{conv}\frac{R_m}{R_{st}c_p}\,. \tag{19}$$

In most cases, the quantity α_{conv} changes little along the surface in the process of cooling of the gas in the case of tube flow or flow across the tube bundle. The heat capacity of the gas changes insignificantly; therefore, in what follows we will consider the quantity β_p to be constant.

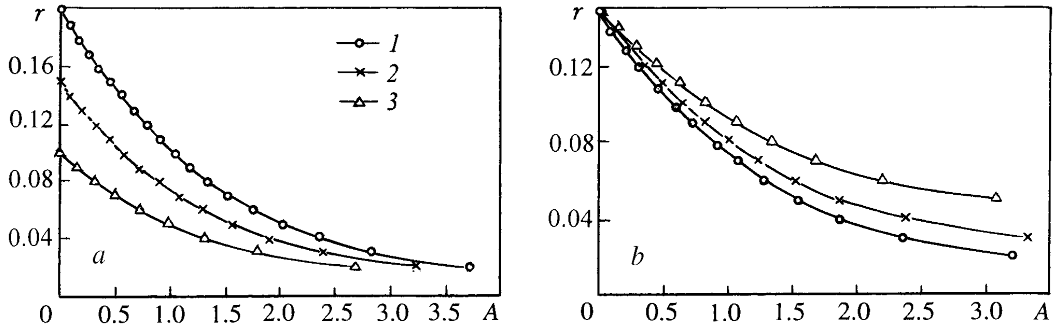

Fig. 2. Relative volume concentration of the steam vs. dimensionless number $A = \alpha_{\text{conv}}F/(c_p m_{\text{dr}} f)$: at $t_{\text{w}} = 10^{\circ}\text{C}$ (a) [1) $r_0 = 0.2$; 2) 0.15; 3) 0.1]; at $r_0 = 0.15$ (b) [1) $t_{\text{w}} = 10$; 2) 20; 3) 30°C].

If the steam-gas flow is considered as an ideal gas, the flow rate of the steam through the cross section f will be

$$G_{\text{st}} = mf\frac{\rho}{\rho_{\text{m}}} = m_{\text{dr}}\frac{r}{1-r}f\frac{R_{\text{m}}}{R_{\text{st}}}, \tag{20}$$

where $R_{\text{st}} = 462$ J/(kg·K).

The molecular mass of the combustion products of natural gas is $\mu_{\text{m}} = r_{\text{N}_2}\mu_{\text{N}_2} + r_{\text{O}_2}\mu_{\text{O}_2} + r_{\text{RO}_2}\mu_{\text{RO}_2} + r\mu_{\text{H}_2\text{O}} = 0.6615\cdot 28 + (0.2509 - r)\cdot 32 + 0.0876\cdot 44 + r\cdot 18 = 30.41 - 14r$, and $R_{\text{m}} = 8314/\mu_{\text{m}} = 594/(2.17 - r)$ J/(kg·K). As r changes from 0.15 to zero, R_{m} decreases from 294.1 to 273.7 J/(kg·K); therefore, as a first approximation it can be considered to be constant and equal to 284 J/(kg·K). Then Eq. (20) will take the form

$$G_{\text{st}} = 0.615 m_{\text{dr}}\frac{r}{1-r}f. \tag{20$'$}$$

Since $m_{\text{dr}} f = \text{const}$, we have

$$dG_{\text{st}} = 0.615 m_{\text{dr}} f\frac{dr}{(1-r)^2}. \tag{21}$$

A comparison of (21) and (15) yields

$$\frac{dr}{(1-r)^2 (r - r_{\text{w}})} = -\frac{\beta_p}{0.615 m_{\text{dr}} f} dF. \tag{22}$$

Integrating the left-hand side of (22) from r_0 to r and the right-hand side from 0 to F respectively, we obtain the change in the relative volume concentration r of the steam along the heat exchange surface F:

$$\frac{dr}{(1-r)^2 (r - r_{\text{w}})} = -\frac{\beta_p}{0.615 m_{\text{dr}} f} dF. \tag{22}$$

Substituting the value of β_p from (19), we will finally have

$$\frac{1}{(1-r_{\text{w}})^2}\ln\frac{(r - r_{\text{w}})(1 - r_0)}{(r_0 - r_{\text{w}})(1 - r)} - \frac{r_0 - r}{(1 - r_{\text{w}})(1 - r)(1 - r_0)} = -\frac{\alpha_{\text{conv}}F}{c_p m_{\text{dr}} f}. \tag{24}$$

The dependence of r on the dimensionless number $A = \alpha_{\text{conv}}/F(c_p m_{\text{dr}} f)$ is presented in Fig. 2.

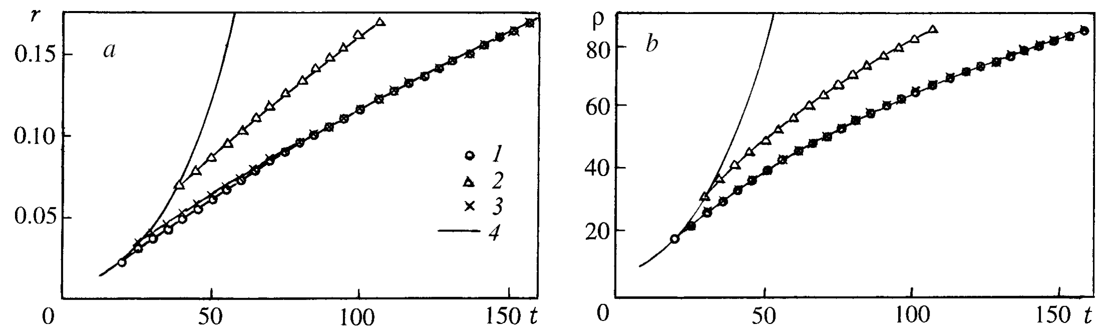

Fig. 3. Change in the relative volume (a) and mass (b) concentrations of the steam in the flow of gases in the process of their cooling: 1) t_0/t_w = 157/17, 2) 107/7, 3) 157/7, and 4) in the saturation state. t, ^{o}C; ρ, g/m^3.

Since, in parallel to the process of mass transfer, the gas is cooled due to the heat transfer to the same surface, dependence (24) holds only until the steam-gas mixture reaches the dew-point temperature.

Change of State of the Steam in the Process of Cooling. At $t > 100^{o}$C, the relative humidity $\varphi = \rho/\rho_{sat}$ of the combustion products in the process of their cooling in the surface heat exchanger decreases because of the condensation of the steam since in this case by ρ_{sat} we mean the density of the saturated steam at barometric pressure (ρ = 0.5977 kg/m^3 at p = 0.101 MPa).

In cooling of the gases below 100^{o}C in the surface heat exchanger, φ will finally increase, reaching unity when the steam-gas mixture is cooled down to a certain temperature. In this connection, it is expedient to find the relationship between the temperature and the humidity of the gas in the process of cooling. The quantity of heat dQ transferred by convection in cooling of the flow is equal, according to the heat-conduction equation, to

$$dQ = \alpha_{conv} (t - t_w)\, dF\,, \tag{25}$$

and, according to the heat-balance equation, to

$$dQ = -\, mfc_p dt = -\, \frac{m_{dr}}{1 - r} fc_p dt\,. \tag{26}$$

Equating (25) and (26) and substituting dF from (15), we obtain

$$\frac{\alpha_{conv} (t - t_w)}{\beta_p (r - r_w)} dG_{st} = \frac{m_{dr}}{1 - r} fc_p dt\,. \tag{27}$$

Taking account of (19) and (21), we have

$$\frac{\alpha_{conv}}{\beta_p} = \frac{c_p}{0.615} \tag{28}$$

and finally write

$$\frac{dr}{(1 - r)(r - r_w)} = \frac{dt}{t - t_w}\,. \tag{29}$$

Integration of the left-hand side from r_0 to r and of the right-hand side from t_0 to t leads, upon a little manipulation, to a dependence of the form

$$\left(\frac{(r - r_w)(1 - r_0)}{(r_0 - r_w)(1 - r)}\right)^{\frac{1}{1 - r_w}} = \frac{t - t_w}{t_0 - t_w}\,. \tag{30}$$

Calculations according to (30) are given in Fig. 3a. In the caption, the figures separated by a fraction bar denote the temperature of the steam-gas mixture at entry to the heat exchanger (in the numerator) and the temperature of the wall (in the denominator). Thus, for example, at $t_w = 7^oC$ the steam contained in the gases reaches the saturation state at $t = 40^oC$ if $t_0 = 107^oC$ and at $t = 25^oC$ if $t_0 = 157^oC$. In deeper cooling, water droplets (mist) appear in the gas flow. Equation (30) for this range of values (to the left of the saturation curve) fails.

For the sake of comparison Fig. 3b gives essentially the same dependence but obtained earlier on the basis of the mass-exchange equation which has been written in the form (1) and not (1′) (disregarding the decrease in the volume of the steam-gas mixture due to the condensation of the steam) [12]:

$$\rho = \rho_0 \frac{T_0}{T} \frac{T - T_w}{T_0 - T_w} + \rho_w \left[\frac{T_w}{T} \frac{T_0 - T}{T_0 - T_w} + \frac{T - T_w}{T} \ln \frac{T_0 - T_w}{T - T_w} \right]. \tag{31}$$

A comparison of the figures shows that under the conditions where the concentration of the steam in the steam-gas mixture is low ($r_0 < 0.175$), both expressions yield coincident results, in particular, virtually the same values of the temperature of the steam-gas mixture at which it becomes saturated in the process of cooling.

CONCLUSIONS

In deep cooling of the combustion products of natural gas in a surface heat exchanger, the steam contained in them is in the superheated state to a temperature which is the lower, the higher the temperature of the products t_0 at entry to the heat exchanger. The reason is the increase in the required heating area with increase in t_0 and hence increase in the surface on which the steam condenses. At $t_0 = 150^oC$ and $t_w < 30^oC$, one can cool the combustion products down to 40 to 50^o C without fearing that the steam in them will become saturated and will condense on the walls of by-passes and a smoke tube. Up to 70% of the steam contained in the combustion products condenses and hence gives up heat in the heat exchanger.

NOTATION

a, thermal diffusivity of the gases, m^2/sec; c_p, heat capacity of the steam-gas mixture. J/(kg·K); D, coefficient of diffusion of the steam in combustion products (air), m^2/sec; d, diameter of the heat-exchanger tube, m; G_{st}, mass flux of the steam per unit time, kg/sec; F, heat-exchange surface, m^2; f, cross-sectional area of the channel, m^2; j, density of the steam-mass flux to the wall, kg/(m^2/sec); m and m_{dr}, mass velocity of the steam-gas mixture and the dry gases, kg/(m^2/sec); Nu, Nusselt number; p, total pressure of the steam-gas mixture, MPa; Pr, Prandtl number; Q, heat flux by convection from the gases to the wall, W; q, density of the heat flux from the gases to the wall, W/ m^2; q_{st}, specific heat of condensation, J/kg; R, gas constant, J/(kg·K); r, relative volume concentration of the steam in the gas flow; Re, Reynolds number; T and t, temperature of the steam in the gas flow at a distance from the wall, K and ^oC; w, velocity of flow of the gases across the tube, m/sec; α, coefficient of heat transfer from the gases to the wall, W/(m^2·K); β, mass-transfer coefficient, m/sec; β_p, mass-transfer coefficient in formula (1′), kg/(m^2·sec); φ, relative humidity; λ, thermal conductivity of the gases, W/(m·K); μ, molar mass, kg/kmole; ν, coefficient of kinetic viscosity of the gases, m^2/sec; Π_d, difference of the relative volume concentrations of the steam at the wall and in the gas flow; ρ, mass concentration of the steam, kg/m^3; ρ_m, density of the steam-gas mixture, kg/m^3. Subscripts: d, diffusion; conv, convection; cond, condensation; sat, saturated; st, steam; m, mixture; w, wall; dr, dry gases; Σ, total; 0, initial conditions, atmospheric pressure; 1, absence of the Stefan flow.

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